Topic: Radius of the balloon

Question: How fast is the radius increasing?

Air is being pumped into a spherical ball at a rate of 2 cm^3/s. What is the rate of increase of the radius of the ball when the radius reaches 10 cm?

Answer choices:

A $\frac{1}{200\pi}$ cm/s

B 400π cm/s

C 200π cm/s

D $\frac{1}{400\pi}$ cm/s

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Solution: A

As with any related rates problem, the first thing we should do is write down what we know.

Change in volume with respect to time $\frac{dV}{dt} = 2$

Volume of the air in the spherical balloon $V = \frac{4}{3}\pi r^3$

And then we write down what we need to find.

Change in radius with respect to time when $r = 10$ $\frac{dr}{dt}$

Since we know we need to solve for dr/dt, we can expect to use implicit differentiation to take the derivative of the equation for volume of the air, since taking the derivative of that equation with respect to time will give us dr/dt on the right.

So, starting with our water volume equation, we'll use implicit differentiation to take the derivative of both sides of the equation with respect to time.

$$V = \frac{4}{3}\pi r^3$$

$$(1)\frac{dV}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

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We know that $dV/dt = 2$ and that $r = 10$, so we'll plug those in and solve for dr/dt.

$$2 = 4\pi(10)^2\frac{dr}{dt}$$

$$2 = 400\pi\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{2}{400\pi}$$

$$\frac{dr}{dt} = \frac{1}{200\pi}$$

This is the rate at which the length of the radius is increasing when the radius is $r = 10$.

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Topic: Radius of the balloon

Question: Air is being pumped into a spherical balloon so that its volume is increasing by 100 cm^3/s. How fast is the radius increasing when the radius is 10 cm?

Answer choices:

A $\frac{1}{4}$ cm/s

B 4 cm/s

C $\frac{1}{4\pi}$ cm/s

D 4π cm/s

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Solution: C

We know that the volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

The derivative of the volume equation with respect to time t is

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

We've been given that $r = 10$ cm and $dV/dt = 100$ cm^3/s.

$$100\frac{\text{cm}^3}{\text{s}} = 4\pi(10\ \text{cm})^2 \frac{dr}{dt}$$

$$100\frac{\text{cm}^3}{\text{s}} = 400\pi\ \text{cm}^2 \frac{dr}{dt}$$

$$\frac{100}{400\pi\ \text{cm}^2} \cdot \frac{\text{cm}^3}{\text{s}} = \frac{dr}{dt}$$

$$\frac{1}{4\pi}\ \text{cm/s} = \frac{dr}{dt}$$

The radius is increasing at a rate of $1/4\pi$ cm/s when the radius is 10 cm.

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Topic: Radius of the balloon

Question: Air is being pumped out of a spherical balloon so that its volume is decreasing by 250 cm^3/s. How fast is the radius decreasing when the radius is 5 cm?

Answer choices:

A $\frac{5}{2\pi}$ cm/s

B $-\frac{5}{2\pi}$ cm/s

C $\frac{2}{5\pi}$ cm/s

D $-\frac{2}{5\pi}$ cm/s

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Solution: A

We know that the volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

The derivative of the volume equation with respect to time t is

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

We've been given that $r = 5$ cm and $dV/dt = -250$ cm^3/s.

$$-250\frac{\text{cm}^3}{\text{s}} = 4\pi(5\text{ cm})^2 \frac{dr}{dt}$$

$$-250\frac{\text{cm}^3}{\text{s}} = 100\pi\text{ cm}^2 \frac{dr}{dt}$$

$$-\frac{250}{100\pi\text{ cm}^2} \cdot \frac{\text{cm}^3}{\text{s}} = \frac{dr}{dt}$$

$$-\frac{5}{2\pi}\text{ cm/s} = \frac{dr}{dt}$$

The radius is decreasing at a rate of $5/2\pi$ cm/s when the radius is 5 cm.

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